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ORIGINAL ARTICLE

ODD MEAN AND EVEN MEAN GRAPHS OF SOME GRAPHS

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ABSTRACT

In this paper we discuss about a new type of labeling known as odd mean labeling and even mean labeling . A graph G with p vertices and q edges is said to have an odd mean labeling if f is an injection from the V(G) to the set {1,3,5,…, 2q -1} such that when each edge uv is assigned the label [f(u)+f(v)]/2, then the resulting edge labels are distinct and even mean labeling if f is an injection from the V(G) to the set {2,4,…, 2q} such that when each edge uv is assigned the label [f(u)+f(v)]/2, then the resulting edge labels are distinct. we prove that the graphs obtained by duplication of a vertex by an edge in cycles of odd length and the ladder graphs admit odd and even mean labeling.

Keywords: Labeling ,Odd mean labeling, Even mean labeling.

1.INTRODUCTION

We begin with simple finite, connected graph G=(V(G),E(G)) with p vertices and q edges. For standard teriminology and notations we follow (Harary, F, 1972). We will provide brief summary of definitions and other informations which are prerequisites for the present investigations.

The concept of mean labeling was introduced by Revathi ,2015 and they investigated the existence of mean labeling for some common graph families.

The concept of odd mean labeling and even mean labeling was introduced by Revathi, (2010) and she proved that umbrella graph, Mangolian tent, K_1+C_n.

Definition 1.1 Duplication of a vertex v_j by a new edge e= $v'_j\, v'_{j+1}$ in a graph G produces a new graph G_1 such that $N(v'_j) \cap N(v'_{j+1}) = v_j$ [Vaidya and Lekha Bijukmar,2010]

Definition 1.2 The ladder graph L_n is a planar undirected graph with *2n* vertices and *n+2(n-1)* edges The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge: $L_{n,} = P_n \times P_2$

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Definition 1.3 . A function f is called an odd mean labeling of a graph G with p vertices and q edges if f is an injection from the vertex set of G to the set {1,3,5,…, 2q-1} such that when each edge uv is assigned the label [f(u)+f(v)]/2, then the resulting edge labels are distinct. A graph which admits an odd mean labeling, is said to be odd mean graph.

Definition 1.4 A function f is called an even mean labeling of a graph G with p vertices and q edges if f is an injection from the vertex set of G to the set {2,4,6,…,2q} such that when each edge uv is assigned the label [f(u)+f(v)]/2, then the resulting edge labels are distinct. A graph which admits an even mean labeling, is said to be even mean graph.

In this paper we investigate odd mean and even mean labeling of graph duplicating a vertex by an edge in cycles of odd length and ladder graph L_n, n>2

2.MAIN RESULT

Theorem 2.1 The graph G obtained by duplication of an arbitrary vertex by a new edge in cycle of odd length, C_{2n-1} ,n>1 is an odd mean graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of a cycle of odd length C_{2n-1} ,n>1 and
let G be the graph obtained by duplicating an arbitrary vertex v_1 of C_{2n-1} , n>1 by a new edge e= $v'_1\, v'_2$.

$E(G) = \{v_1 v'_1, v'_1 v'_2, v_1 v'_2\} \cup \{v_i v_{i+1} \,/\, 1 \le i \le n-1\} \cup \{v_1 v_n\}$

$|V(G)| = n+2, \; n \ge 3$

$|E(G)| = n+3,\ n \geq 3$

Define a labeling $f : V(G) \rightarrow \{1,3,5,\ldots, 2q-1\}$ by

$f(v_1^{'}) = 1,\ f(v_2^{'}) = 3,$

$f(v_i) = 2i+3$ for $1 \leq i \leq n$,

The resulting edge labels are

$f(v_1 v_1^{'}) = [f(v_1)+f(v_1^{'})]/2$
$= (5+1)/2$
$=3$

$f(v_1^{'} v_2^{'}) = [f(v_1^{'})+f(v_2^{'})]/2$
$= (1+3)/2$
$= 2$

$f(v_1 v_2^{'}) = [f(v_1)+f(v_2^{'})]/2$
$= (5+3)/2$
$= 4$

$f(v_iv_{i+1}) = [f(v_i)+f(v_{i+1})]/2$
$= [2i+3+2(i+1)+3]/2$
$= 2i+4$ for $1 \leq i \leq n-1$.

$f(v_1v_n) = [f(v_1)+f(v_n)]/2$
$= (5+2n+3)/2$
$=n+4$

$n+4 \neq 2,3,4$ and $2i+4$ for $1 \leq i \leq n-1$.

thus f admits an odd mean labeling

Hence G is an odd mean graph.

Theorem 2.2 The graph G obtained by duplication of an arbitrary vertex by a new edge in cycle of odd length, C_{2n-1} , n>1 is an even mean graph.

Proof: Let $v_1,v_2,\ldots,v_n$ be the vertices of a cycle of odd length C_{2n-1} , n>1 and let G be the graph obtained by duplicating an arbitrary vertex v_1 of C_{2n-1} , n>1 by a new edge $e= v_1^{'} v_2^{'}$.

$E(G) = \{v_1 v_1^{'}, v_1^{'} v_2^{'}, v_1 v_2^{'}\} \cup \{v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1v_n\}$

$|V(G)| = n+2,\ n \geq 3.$

$|E(G)| = n+3,\ n \geq 3.$

Define a labeling $f : V(G) \rightarrow \{2,4,6,\ldots,2q\}$ by

$f(v_1^{'}) = 2,\ f(v_2^{'}) = 4,$

$f(v_i) = 2i+4$ for $1 \leq i \leq n$,

The resulting edge labels are

$f(v_1 v_1^{'}) = [f(v_1)+f(v_1^{'})]/2$
$= (6+2)/2$
$= 4$

$f(v_1^{'} v_2^{'}) = [f(v_1^{'})+f(v_2^{'})]/2$
$= (2+4)/2$
$= 3$

$f(v_1 v_2^{'}) = [f(v_1)+f(v_2^{'})]/2$
$= (6+4)/2$
$= 5$

$f(v_iv_{i+1}) = [f(v_i)+f(v_{i+1})]/2$
$= [2i+4+2(i+1)+4]/2$
$= 2i+5$ for $1 \leq i \leq n-1$.

$f(v_1v_n) = [f(v_1)+f(v_n)]/2$
$= (6+2n+4)/2$
$=n+5$

$\neq 3,4,5$ and $2i+5$ for $1 \leq i \leq n-1$

thus f admits even mean labeling

Hence G is an even mean graph.

Theorem 2.3 The ladder graph L_n is an odd mean graph if n>2

Proof: Let $G = L_n$ be the ladder graph

$V(G) = \{u_1,u_2,\ldots,u_n, v_1,v_2,\ldots,v_n\}$

$|V(G)| = 2n$

$E(G) = \{u_iv_i /, 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1} /, 1 \leq i \leq n-1\}$

$|E(G)| =3n-2,\ n>2$

Define a labeling $f: V(G) \rightarrow \{1,3,5,\ldots, 2q-1\}$ by

$f(v_1) = 1,\ f(v_2) = 3$

$f(v_i) = 4i-1$ for $3 \leq i \leq n$

$f(u_i) = 2i+3$ for $1 \leq i \leq 3$

$f(u_i) = 4i-3$ for $4 \leq i \leq n$

The resulting edge labels are

$f(v_1v_2) = [f(v_1)+f(v_2)]/2$
$= 1+3)/2$
$= 2$

$f(v_2v_3) = [f(v_2)+f(v_3)]/2$
$= (3+11)/2$
$=7$

$f(v_iv_{i+1}) = [f(v_i)+f(v_{i+1})]/2$
$= [4i-1+4(i+1)-1]/2$
$= 4i+1$ for $3 \leq i \leq n-1$

$f(u_iu_{i+1}) = [f(u_i)+f(u_{i+1})]/2$
$= [2i+3+2(i+1)+3]/2$
$= 2i+4$ for $i=1,2$

$f(u_3u_4) = [f(u_3)+f(u_4)]/2$
$= (9+13)/2$
$= 11$

$f(u_iu_{i+1}) = [f(u_i)+f(u_{i+1})]/2$
$= [4i-3+4(i+1)-3]/2$
$= 4i-1$ for $4 \leq i \leq n-1$
$\neq 2i+4$ for $i=1,2$

$f(u_1v_1) = [f(u_1)+f(v_1)]/2$
$= (5+1)/2$
$=3$

$f(u_2v_2) = [f(u_2)+f(v_2)]/2$
$= (7+3)/2$
$=5$

$f(u_3v_3) = [f(u_3)+f(v_3)]/2$
$= (9+11)/2$
$= 10$

$f(u_iv_i) = [f(u_i)+f(v_i)]/2$
$= [4i-3+4i-1]/2$
$= 4i-2$ for $4 \leq i \leq n$.

thus f admits an odd mean labeling

Hence G is an odd mean graph.

Theorem 2.4 The ladder graph L_n is an even mean graph n>2 .

Proof: Let $G = L_n$ be the ladder graph

$V(G) = \{u_1,u_2,\ldots,u_n, v_1,v_2,\ldots,v_n\}$

$|V(G)| = 2n$

$E(G) = \{u_iv_i / 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1} /, 1 \leq i \leq n-1\}$

$|E(G)| = 3n-2,\ n>2$

Define a labeling $f: V(G) \rightarrow \{2,4,\ldots,2q\}$ by

$f(v_1) = 2,\ f(v_2) = 4$

$f(v_i) = 4i$ for $3 \leq i \leq n$

$f(u_i)$ = 2i+4 for $1 \leq i \leq 3$
$f(u_i)$ = 4i-2 for $4 \leq i \leq n$
The resulting edge labels are
$f(v_1v_2) = [f(v_1)+f(v_2)]/2$
$= (2+4)/2$
$= 3$
$f(v_2v_3) = [f(v_2)+f(v_3)]/2$
$= (4+12)/2$
$= 8$
$f(v_iv_{i+1}) = [f(v_i)+f(v_{i+1})]/2$
$= [4i+4(i+1)]/2$
$= 4i+2$ for $3 \leq i \leq n$-1.
$f(u_iu_{i+1}) = [f(u_i)+f(u_{i+1})]/2$
$= [2i+4+2(i+1)+4]/2$
$= 2i+5$ for i=1,2
$f(u_3u_4) = [f(u_3)+f(u_4)]/2$
$= (10+14)/2$
$=12$
$f(u_iu_{i+1}) = [f(u_i)+f(u_{i+1})]/2$
$= [4i-2+4(i+1)-2]/2$
$= 4i$ for $4 \leq i \leq n$-1.
$\neq 2i+5$ for i=1,2
$f(u_1v_1)$) $= [f(u_1)+f(v_1)]/2$
$= (6+2)/2$
$= 4$
$f(u_2v_2) =[f(u_2)+f(v_2)]/2$
$=(8+4)/2$
$=6$
$f(u_3v_3) = [f(u_3)+f(v_3)]/2$
$= (10+12)/2$
$= 11$
$f(u_iv_i) = [f(u_i)+f(v_i)]/2$
$= [4i-2+4i]/2$
$= 4i-1$ for $4 \leq i \leq n$
thus f admits even mean labeling
Hence G is an even mean graph.

3.REFERENCES

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