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ORIGINAL ARTICLE

TRUNCATED MOMENT'S EFFECTS IN CHARACTERIZATION OF SKEW NORMAL DISTRIBUTION

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ABSTRACT

A probability distribution can be characterized through various methods.This paper discusses a truncated moment's effects in characterization of skew normal distribution by considering a product of reverse hazard rate and another function of the truncated point.These characterizations may also serve as a basis for parameter estimation.It is hoped that the findings of the paper will be useful for researchers in different fields of applied sciences.

Keywords: Emergence a truncated moment's effects, characterization of skew normal distribution

1.INTRODUCTION

The characterization of a probability distribution plays an important role in probability and statistics.A probability distribution can be characterized through various methods, The development of the general theory of the characterization of probability distributions by truncated moment began with the work of Galambos and Kotz (1978). Further development in this area continued with the contributions of many authors and researchers , among them Kotz and Shanbhag (1980), Glanzel(1987,1990), and Glanzel etal(1984), are notable.However the most of these characterizations are based on a simple relationship between two different moments truncated from the left at the same point.In this paper, I present a new characterization of the skew normal distribution using the truncated moment by considering a product of reverse hazard rate and another function of the truncated point.

This paper is as follows:
Section 2 : discuss the skew normal distribution(SND) and some of its properties.
Section 3: characterization of the skew normal distribution by truncated moment is presented.
Section 4 : The concluding remarks are provided.

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2.DEFINITION:

A continuous random variable X~ (μ ,σ, λ) is said to have a skew normal distribution ,denoted by $Y \sim SN(\mu ,\sigma^2 ,\lambda)$, if its probability density function and cumulative distribution function are respectively, given by

$$f(x ,\mu ,\sigma ,\lambda) = 2\phi\left[\frac{x-\mu}{\sigma}\right]\Phi[\lambda(\frac{x-\mu}{\sigma})] , \quad (2.1)$$

$$F(x,\mu,\sigma,\lambda)=\Phi\left[\frac{x-\mu}{\sigma}\right]-2T[\frac{x-\mu}{\sigma},\lambda] , \quad (2.2)$$

where $-\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$ and $-\infty < \lambda < \infty$, are to be referred as the location , scale and shape parameters, respectively. ϕ(u) and Φ(λu) denote the probability density function and cumulative distribution function of the standard normal distribution, respectively; and T(u,λ) denotes Owen's (1956) T function as given by,

$$T(u,\lambda) = \frac{1}{2\pi}\int_0^{\lambda} \frac{e^{-\left(\frac{1}{2}\right)u^2(1+x^2)}}{1+x^2}\,dx.$$

In particular, if in the above definitions μ= 0, σ = 1, then we have a standard skew normal distribution , denoted by $X_\lambda \sim SN(\lambda)$, with the probability density function as given by,

$f_X(x;\lambda) = 2\phi(x)\,\Phi(\lambda x), -\infty < x < \infty,$ (2.3)

where $\phi(x) = [\frac{1}{\sqrt{2\pi}}]e^{\frac{-1}{2}x^2}$

and $\Phi(\lambda x) = \int_{-\infty}^{\lambda x} \phi(t)dt$

denote the probability density function and cumulative distribution function of the standard normal distribution respectively.

Some characteristic values of the random variable Y_λ are as follows:

Mean: $E(Y_\lambda) = \mu + [\sigma\delta\sqrt{\frac{2}{\pi}}]$

Variance: $Var(Y_\lambda) = \frac{\sigma^2(\pi-2\delta^2)}{\pi}$, and

$MGF = 2e^{\lambda t+\frac{\sigma^2 t^2}{2}}\Phi(\lambda\sigma t)$.

Skewness: $\gamma_1 = (\frac{4-\pi}{2})\frac{[E(X_\lambda)]^3}{[var(X_\lambda)]^{\frac{3}{2}}} = \frac{4-\pi}{2}(\delta\sqrt{2/\pi})^3(1-\frac{2\delta^2}{\pi})^{-3/2}$, where $\delta = \frac{\lambda}{1+\lambda^2}$.

Kurtosis: $\gamma_2 = 2(\pi-3)\frac{[E(X_\lambda)]^4}{[var(X_\lambda)]^2} = 2(\pi-3)(\delta\sqrt{2/\pi})^4(1-\frac{2\delta^2}{\pi})^{-2}$.

Note: The skewness is limited in the interval(-1,1).

The shape of the skew normal probability density function given by(2.1) depends on the values of the parameter λ. For some values of the parameters (μ ,σ, λ), the shapes of the pdf of SN(λ) (2.1) are provided in Figure 1 below.

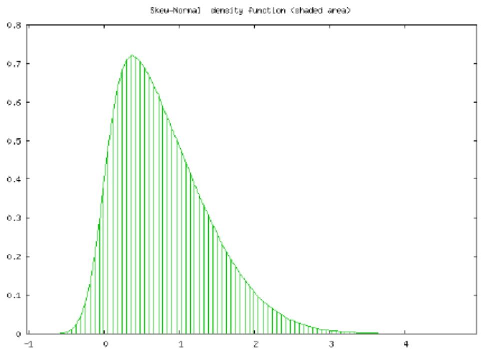

Figure 1: Plot of the pdf of SN(λ) for $\mu = 0, \sigma = 1, \lambda = 5$

The skew normal distribution represents a parametric class of probability distributions, reflecting varying degrees of skewness, which includes the standard normal distribution as a special case. The term skew normal distribution (SND) was introduced by Azzalini(1985,86), who gave a systematic treatment of this distribution.

A NEW CHARACTERIZATION OF THE SKEW NORMAL DISTRIBUTION

we need the following assumptions and Lemma (Lemma3.1). Let X be a random variable having absolutely continuous (with respect to Lebesgue measure)cumulative distribution function(cdf) F(x) and the probability density function (pdf) f(x).We assume α =inf{x/F(x)>0} and β = sup{x/F(x)<1}. We define

$\eta(x) = \frac{f(x)}{F(x)}$,and g(x) is a differential function with respect to x for all real x ∈ (α ,β).

Lemma 3.1

Suppose that X has an absolutely continuous (with respect to Lebesgue measure) cdf F(x), with corresponding pdf f(x) and E(X\ X≤ x) exists for all real x∈(α ,β). Then

$E(X\backslash X\le x) = g(x)\ \eta(x),$

Where g(x) is a differentiable function , and $\eta(x) = \frac{f(x)}{F(x)}$, for all real x∈(α ,β) if

$f(x) = c\,e^{\int_a^x \frac{u-g'(u)}{g(u)}du}$,

where c is determined such that $\int_{-\infty}^{\infty} f(x)dx = 1$.

A characterization theorem

Theorem 3.1:

Suppose that X has an absolutely continuous (with respect to Lebesgue measure) cdf F(x), pdf f(x) and E(X / X ≤ x) exists for all real x ∈ (α, β). We assume α = –∞, β = ∞ and E(X) and f '(x) exist for all x∈(-∞,∞). Then,

$E(X/X\le x) = g(x)\ \eta(x),$

Where, $\eta(x) = \frac{f(x)}{F(x)}$

$g(x) = \frac{x(\Phi(x)-2T(x,\lambda))}{2\phi(x)\Phi(\lambda x)} - \frac{H(x,\lambda)}{2\phi(x)\Phi(\lambda x)}$.

$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$,

$\Phi(x) = \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{\frac{-u^2}{2}}du$

T (x,λ) is the Owen (1956) T function as given by

$T(x,\lambda) = \frac{1}{2\pi}\int_0^{\lambda}\frac{e^{-(\frac{1}{2})x^2(1+u^2)}}{1+u^2}du$

and $H(x,\lambda) = \int_{-\infty}^{x}(\Phi(u) - 2T(u,\lambda))du$,

if and only if

$f(x) = 2\phi(x)\,\Phi(\lambda x)$,for any real λ.

Proof:

Suppose $f(x) = 2\phi(x)\,\Phi(\lambda x)$,

then $g(x) = \frac{\int_{-\infty}^{x}uf(u)du}{f(x)} = \frac{xF(x)}{f(x)} - \frac{\int_{-\infty}^{x}F(x)}{f(x)}$

$$= \frac{x[\Phi(x) - 2T(x,\lambda)]}{\phi(x)\Phi(\lambda x)} - \frac{H(x,\lambda)}{\phi(x)\Phi(\lambda x)}$$

Suppose that

$$g(x) = \frac{x(\Phi(x) - 2T(x,\lambda))]}{2\phi(x)\Phi(\lambda x)} - \frac{H(x,\lambda)}{2\phi(x)\Phi(\lambda x)}$$

then

$$g'(x) = x + [\frac{x(\Phi(x) - 2T(x,\lambda))]}{2\phi(x)\Phi(\lambda x)} - \frac{H(x,\lambda)}{2\phi(x)\Phi(\lambda x)}](x - \frac{\lambda\phi(\lambda x)}{\Phi(\lambda x)})$$

$$= x + g(x)[x - \frac{\lambda\varphi(\lambda x)}{\Phi(\lambda x)}].$$

Thus,

$$\frac{f'(x)}{f(x)} = \frac{x - g'(x)}{g(x)} = -x + \lambda\frac{\phi(\lambda x)}{\Phi(\lambda X)}.$$

On integrating the above equation, we have

$$f(x) = ce^{-\left(\frac{1}{2}\right)x^2}\Phi(\lambda x),$$

where, $1/c = \int_{-\infty}^{\infty} e^{-\left(\frac{1}{2}\right)x^2}\Phi(\lambda x)\,dx = \frac{\sqrt{2\pi}}{2}$.

Hence the proof.

3. CONCLUDING REMARKS

1. The, characterization of a probability distribution plays an important role in probability and statistics .
2. This paper considers a new characterization of the skew normal distribution using truncated moment by considering a product of reverse hazard rate and another function of the truncated point.
3. In this regard, some distributional properties of the skew normal distribution are also provided.
4. I believe that the findings of this paper would be useful the practitioners in various fields of studies and further enhancement of research in distribution theory, and its applications.

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