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ORIGINAL ARTICLE

A STUDY ON ODD MEAN AND EVEN MEAN LABELING OF SPECIAL GRAPH FAMILIES

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ABSTRACT

A graph $G=(V(G),E(G))$ with p vertices and q edges is said to be an odd mean graph if it is possible to label the vertices $v\in V(G)$ with distinct label $f(v)$ from $\{1,3,5,....,2q-1\}$ in such a way that when each edge is labelled with $f(uv)=\dfrac{f(u)+f(v)}{2}$ then the resulting edge labels are distinct. A graph $G=(V(G),E(G))$ with p vertices and q edges is said to be an even mean graph if it is possible to label the vertices $v\in V(G)$ with distinct label $f(v)$ from $\{2,4,6,....,2q\}$ in such a way that when each edge is labelled with $f(uv)=\dfrac{f(u)+f(v)}{2}$ then the resulting edge labels are distinct. In this paper, we prove that for all odd number $n>1$, the $n-$sunlet graph is both odd and even mean graph. Also, we study the odd mean and even mean labeling of the special graph L_nAK_1.

Keywords: Odd Mean Labeling, Even Mean Labeling

1.INTRODUCTION

For all definition, terminologies and notations not specifically defined in this paper, we refer(Gallian, 2014, Harary, 1972). Unless otherwise mentioned, all graphs considered here are simple, finite and have no isolated vertices.

The concept of odd mean labeling and even mean labeling was introduced by Revathi, (2015) and she proved that the umbrella graph, Mangolian Tent and K_1+C_n are admit odd mean labeling as well as even mean labeling. The same labeling techniques were investigated for the graph obtained by duplication of an arbitrary vertex by a new edge in a cycle of odd length C_{2n+1} and for the ladder graph L_n by Santhanaraju and Meena, (2016). The harmonic mean labeling was studied by Sandhya and Somasundaram, (2013) for the graph L_nAK_1.

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In this paper, we study the odd mean and even mean labeling of the special graph families the $n-$sunlet graph and L_nAK_1. We will give all prerequisite of the present study.

Definition 1.1 A graph obtained by attaching a pendent edge at each vertex of a cycle C_n is called $n-$sunlet graph.

Definition 1.2 The graph obtained by attaching a pendent edge at each vertex of a ladder graph L_n is called L_nAK_1-graph.

Definition 1.3 A graph $G=(V(G),E(G))$ with p vertices and q edges is said to be an odd mean graph if there is an injection $f:V(G)\to\{1,3,5,....,2q-1\}$ such that each edge uv is assigned a label $\dfrac{f(u)+f(v)}{2}$ then the resulting edge labels are distinct. In this case, we say that f admits an odd mean labeling.

Definition 1.4 A graph $G=(V(G),E(G))$ with p vertices and q edges is said to be an even mean graph if there is an injection $f:V(G)\to\{2,4,6,....,2q\}$ such that each edge uv is assigned a label $\frac{f(u)+f(v)}{2}$ then the resulting edge labels are distinct. In this case, we say that f admits an even mean labeling.

2. MAIN RESULTS

Theorem 2.1 The $n-$sunlet graph is an odd mean graph for all odd number $n>1$.

Proof: Let G be a $n-$sunlet graph.

Let $V(G)=\{v_1,v_2,v_3,....,v_{2n}\}$

$|V(G)|=2n$

$E(G)=\{v_iv_{i+1}\,|1\le i\le n\}\cup\{v_1v_{n+2},v_2v_{n+3},....,v_{n-1}v_{2n}\}\cup\{v_1v_n\}$

$|E(G)|=2n.$

The vertex labeling $f:V(G)\to\{1,3,5,....,4n-1\}$ is defined by

$f(v_i)=2i-1,\ 1\le i\le 2n.$

The resulting edge labels are

$$f(v_iv_{i+1})=\frac{f(v_i)+f(v_{i+1})}{2}$$
$$=\frac{2i-1+2(i+1)-1}{2}$$
$$=2i,1\le i\le n$$
$$f(v_iv_{n+i+1})=\frac{f(v_i)+f(v_{n+i+1})}{2}$$
$$=\frac{2i-1+2(n+i+1)-1}{2}$$
$$=2i+n,\ 1\le i\le n-1$$
$$f(v_1v_n)=\frac{f(v_1)+f(v_n)}{2}$$
$$=\frac{1+2n-1}{2}$$
$$=n$$

Thus f admits an odd mean labeling and hence the $n-$sunlet graph is an odd mean graph for all odd number $n>1$.

Theorem 2.2 The $n-$sunlet graph is an even mean graph for all odd number $n>1$.

Proof: Let G be a $n-$sunlet graph.

Let $V(G)=\{v_1,v_2,v_3,....,v_{2n}\}$

$|V(G)|=2n$

$E(G)=\{v_iv_{i+1}\,|1\le i\le n\}\cup\{v_1v_{n+2},v_2v_{n+3},....,v_{n-1}v_{2n}\}\cup\{v_1v_n\}$

$|E(G)|=2n.$

The vertex labeling $f:V(G)\to\{2,4,6,....,4n\}$ is defined by

$f(v_i)=2i,\ 1\le i\le 2n.$

The resulting edge labels are

$$f(v_iv_{i+1})=\frac{f(v_i)+f(v_{i+1})}{2}$$
$$=\frac{2i+2(i+1)}{2}$$
$$=2i+1,\ 1\le i\le n$$
$$f(v_iv_{n+i+1})=2i+n+1,\ 1\le i\le n-1$$
$$f(v_1v_n)=n+1$$

Thus f admits an even mean labeling and hence the $n-$sunlet graph is an even mean graph for all odd number $n>1$.

Theorem 2.3 The graph L_nAK_1 is an odd mean graph for all $n>1$.

Proof: Let G be a L_nAK_1 graph for $n>1$.

Let $V(G)=\{v_1,v_2,v_3,....,v_{4n}\}$

$|V(G)|=4n$

$E(G)=\{v_iv_{i+1}\,|1\le i\le 4n-1\ and\ i\ne 4k,\ k=1,2,3,....,n-1\}\cup\{v_{4i-2}v_{4i+2},v_{4i-1}v_{4i+3}\,|1\le i\le n-1\}$

$|E(G)|=5n-2.$

The vertex labeling $f:V(G)\to\{1,3,5,....,8n-1\}$ is defined by

$f(v_i)=2i-1,\ 1\le i\le 4n.$

The resulting edge labels are

$$f(v_iv_{i+1})=\frac{f(v_i)+f(v_{i+1})}{2}$$
$$=\frac{2i-1+2(i+1)-1}{2}$$
$$=2i,\ 1\le i\le 4n-1\ and\ i\ne 4k,k=1,2,3,....,n-1$$
$$f(v_{4i-2}v_{4i+2})=\frac{f(v_{4i-2})+f(v_{4i+2})}{2}$$
$$=\frac{2(4i-2)-1+2(4i+2)-1}{2}$$
$$=8i-1,\ 1\le i\le n-1$$
$$f(v_{4i-1}v_{4i+3})=\frac{f(v_{4i-1})+f(v_{4i+3})}{2}$$
$$=\frac{2(4i-1)-1+2(4i+3)-1}{2}$$
$$=8i+1,\ 1\le i\le n-1$$

Thus f admits an odd mean labeling and hence the L_nAK_1 graph is an odd mean graph for all $n>1$.

Theorem 2.4 The graph $L_n AK_1$ is an even mean graph for all $n > 1$.

Proof: Let G be a $L_n AK_1$ graph for $n > 1$.

Let $V(G) = \{v_1, v_2, v_3, ..., v_{4n}\}$

$|V(G)| = 4n$

$E(G) = \{v_i v_{i+1} \mid 1 \le i \le 4n-1 \text{ and } i \ne 4k, k = 1,2,3,....,n-1\} \cup \{v_{4i-2}v_{4i+2}, v_{4i-1}v_{4i+3} \mid 1 \le i \le n-1\}$

$|E(G)| = 5n-2.$

The vertex labeling $f : V(G) \to \{2,4,6,....,8n\}$ is defined by

$f(v_i) = 2i,\ 1 \le i \le 4n.$

The resulting edge labels are

$$f(v_i v_{i+1}) = \frac{f(v_i) + f(v_{i+1})}{2}$$
$$= \frac{2i + 2(i+1)}{2}$$
$$= 2i+1,\ 1 \le i \le 4n-1 \text{ and } i \ne 4k, k = 1,2,3,....,n-1$$

$$f(v_{4i-2}v_{4i+2}) = \frac{f(v_{4i-2}) + f(v_{4i+2})}{2}$$
$$= \frac{2(4i-2) + 2(4i+2)}{2}$$
$$= 8i,\ 1 \le i \le n-1$$

$$f(v_{4i-1}v_{4i+3}) = \frac{f(v_{4i-1}) + f(v_{4i+3})}{2}$$
$$= \frac{2(4i-1) + 2(4i+3)}{2}$$
$$= 8i+2,\ 1 \le i \le n-1$$

Thus f admits an even mean labeling and hence the $L_n AK_1$ graph is an even mean graph for all $n > 1$.

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