Available online at www.journalijmrr.com

INTERNATIONAL JOURNAL OF MODERN RESEARCH AND REVIEWS

Int. J. Modn. Res. Revs.

Volume 8, Issue 1, pp 1-6, January, 2020

ISSN: 2347-8314

CUBE DIFFERENCE MEAN LABELING OF GRAPHS

[*1]G.Sudarvizhi and [2]K.Balasangu

[*1]Assistant Professor, Department of Mathematics, Govt Arts College , Chidambaram - 608102, Tamil Nadu , India.

[2]Assistant Professor, Department of Mathematics, T.K. Govt Arts College , Virudhachalam - 606001, Tamil Nadu , India.

Article History: *Received 20nd January,2020, Accepted 30th January, 2020, Published 31st January,,2020*

ABSTRACT

A graph G = (V,E) with p vertices and q edges is said to be cube difference mean labeling, if their exist a bijection $f: V(G) \to \{1,2,3,\dots,q+1\}$ such that the induced function $f^*(uv) = E(G) \to \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$ given by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$ for every $uv \in E(G)$ are all distinct. A graph which admits Cube Difference Mean Labeling is called Cube Difference Mean Graph. In this paper we proved that some classes of graphs like, path, Comb and Y-Tree(Y_n) graphs are Cube Difference Mean Labeling Graphs.

Keywords: Cube Difference Mean Labeling, Cube Difference Mean graph, P_n, Comb graph and Y-Tree(Y_n).

1.INTRODUCTION

All graphs in this paper are simple finite undirected and nontrivial graph G = (V,E) with vertex set V and the edge set E. For graph theoretic terminology, we refer to Harary [1972]. A dynamic survey on graph labeling is regularly updated by Gallian [2003] and it is published by Electronic Journal of combinatorics. The concept of mean labeling was introduced and studied by Somasundaram and Ponraj [2003]. Further some more results on mean graphs are discussed by Somasundaram and Ponraj [2003] . Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. The Square Difference Labeling is previously defined by Ajitha et al., [2006].. The concept of Cube Difference Labeling was first introduced by Shiama [2013]and it was proved in Shiama [2013 that many standard graphs. Further some more references of [Amuda and Meena 2015 ;Gowri and Vembarasi. 2017; Sandhya et al.,2015; Sandhya and Somasundaram,2013;] Gayathri and Prakash ,2015]. Motivated by the work of these authors, We Introduce Cube difference Mean labeling graphs, and also in this paper we investigate Cube Difference Mean Labeling of Path, comb and Y-Tree (Y_n) .

1.1 Definition

A graph G = (V,E) with p vertices and q edges is said to be cube difference mean labeling, if their exist a bijection

Corresponding author:* **G.Sudarvizhi, Assistant Professor, Department of Mathematics, Govt Arts College , Chidambaram - 608102, Tamil Nadu, India.

$f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ such that the induced function $f^*(uv) = E(G) \rightarrow \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$ given by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$ for every $uv \in E(G)$ are all distinct. A graph which admits Cube Difference Mean Labeling is called Cube Difference Mean Graph.

1.2 Definition

A walk in which vertices are distinct is called a path. A path on 'n' vertices is denoted by P_n.

1.3 Definition

The graph obtained by joining a single pendant edge to each vertex of a path is called a comb graph

1.4 Definition

A graph that contains no cycles is called an acyclic graph. A connected acyclic graph is called a tree.

1.5 Definition

A Y-tree is a graph obtained from a path by appending an edge to a vertex of a path adjacent to an end point and it is denoted by Y_n , where 'n' is the number of vertices in the tree.

Theorem 1

Any path P_n is a Cube Difference Mean Labeling Graphs.

Proof:

Let G = P_n be the graph with vertices $v_1, v_2, \dots, v_n$ and the edges , $e_2, \dots, e_{n-1}$. Which are denoted in Fig(1.1)

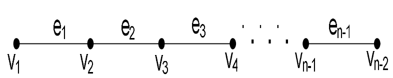

Fig. 1.1

Define the function, $f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ as follows

$f(v_i) = i$ for $1 \le i \le n$

and the induced edge labeling function $f^*(uv) = E(G) \rightarrow \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$ is defined by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$

Now, $f^*(v_{i+1}v_i) = \left\lceil \frac{3i^2+3i+1}{2} \right\rceil$ for $1 \le i \le n$

Clearly $f^*(v_n v_{n-1}) = \left\lceil \frac{3n^2-3n+1}{2} \right\rceil$

Hence the edge labels are distinct.

Thus path graph admits a Cube Difference Mean Labeling Graphs ,

Example 1.1

The following example shows that P_7 is Cube Difference Mean Labeling Graph.

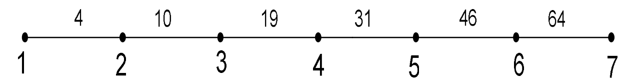

Fig.1.2

Here

$f(v_1) = 1; f(v_2) = 2; f(v_3) = 3; f(v_4) = 4; f(v_5) = 5; f(v_6) = 6;\ f(v_7) = 7$ and

$f^*(v_2v_1) = 4; f^*(v_3v_2) = 10; f^*(v_4v_3) = 19; f^*(v_5v_4) = 31; f^*(v_6v_5) = 46;\ f^*(v_7v_6) = 64$

Hence the edge labeling of P_7 are distinct.

Thus the path P_7 is Cube Difference Mean Labeling Graph.

Theorem 2

Comb graph is Cube Difference Mean Labeling Graph

Proof:

Let G be a comb graph with vertices

$V(G) = \{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_n\}$

Let us take P_n as $v_1, v_2, \dots, v_n$ and join a vertex u_i *to* v_i for $1 \le i \le n$

And the edges are $E(G) = \{e_1, e_2, \dots, e_{n-1}; a_1, a_2, \dots, a_n\}$

Which are denoted in the following Fig(1.3)

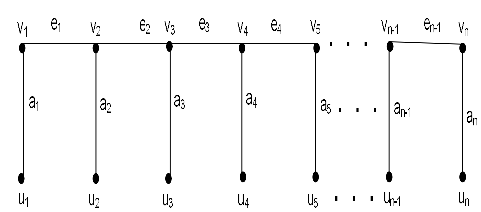

Fig.1.3

Define the function , $f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ as follows

$f(u_i) = 2i$ for $1 \le i \le n$

$f(v_i) = 2i - 1$ for $1 \le i \le n$

Induced edge labeling function is defined by

$f^*(uv) = E(G) \rightarrow \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$ is defined by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$

Now, $f^*(v_{i+1}v_i) = \lceil 12i^2 + 1 \rceil$ for $1 \le i \le n$-1

$f^*(u_i v_i) = \left\lceil \frac{12i^2-6i+1}{2} \right\rceil$ for $1 \le i \le n$

Clearly $f^*(v_n v_{n-1}) = \lceil 12n^2 - 24n + 13 \rceil$

$f^*(u_n v_n) = \left\lceil \frac{12n^2-6n+1}{2} \right\rceil$

Hence the edge labels are distinct.

Thus comb graph admits a Cube Difference Mean Labeling Graph.

Example 2.1

Cube difference mean labeling of comb graph obtained from P_6 is given below.

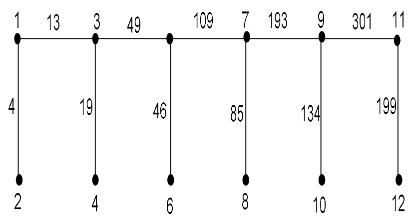

Fig.1.4

Here, the vertex labels are,

$f(v_1) = 1; f(v_2) = 3; f(v_3) = 5; f(v_4) = 7; f(v_5) = 9; f(v_6) = 11$ and

$f(u_1) = 2; f(u_2) = 4; f(u_3) = 6; f(u_4) = 8; f(u_5) = 10; f(u_6) = 12$

Edge labels are,

$f^*(v_2 v_1) = 13; f^*(v_3 v_2) = 49; f^*(v_4 v_3) = 109; f^*(v_5 v_4) = 193; f^*(v_6 v_5) = 301$ *and*

$f^*(u_1 v_1) = 4; f^*(u_2 v_2) = 19; f^*(u_3 v_3) = 46; f^*(u_4 v_4) = 85; f^*(u_5 v_5) = 134;$ &

$f^*(u_6 v_6) = 134$

Hence the edges are distinct.

Theorem 3

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two of a comb graph. Then G is Cube Difference Mean Labeling Graph.

Proof:

Let the vertex set of a comb be

$V(G) = \{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_n\}$

Let us take P_n as $v_1, v_2, \dots, v_n$ and join a vertex u_i *to* v_i for $1 \le i \le n$

Let G be graph obtained by joining a pendant vertex w to v_n

And the edges are $E(G) = \{e_1, e_2, \dots, e_n; a_1, a_2, \dots, a_n\}$

Which are denoted as in Fig(1.5)

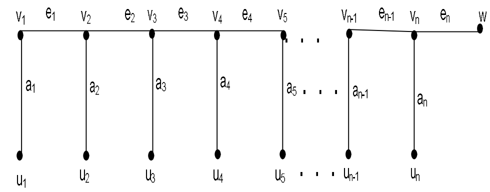

Fig.1.5

Define the function $f: V(G) \to \{1,2,3, \dots, q+1\}$ as follows

$f(u_i) = 2i$ for $1 \le i \le n$

$f(v_i) = 2i - 1$ for $1 \le i \le n$

$f(w) = 2n + 1$

Induced edge labeling function

$f^*(uv) = E(G) \to \{a/1 \le a \le N \text{ and } a \equiv 1(mod 3)\}$ is defined by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$

Now, $f^*(v_{i+1} v_i) = \lceil 12i^2 + 1 \rceil$ for $1 \le i \le n$-1

$f^*(u_i v_i) = \left\lceil \frac{12i^2-6i+1}{2} \right\rceil$ for $1 \le i \le n$

Clearly $f^*(v_n v_{n-1}) = \lceil 12n^2 - 24n + 13 \rceil$

$f^*(u_n v_n) = \left\lceil \frac{12n^2-6n+1}{2} \right\rceil$

$f^*(w v_n) = \lceil 12n^2 + 1 \rceil$

Hence the edge labels are distinct.

This gives Cube Difference Mean Labeling Graph.

Example 3.1

Cube difference mean labeling G with 11 vertices 10 edges is shown below:

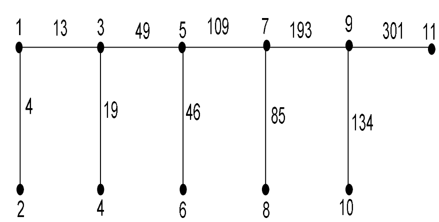

Fig.1.6

Here, the vertex labels are,

$f(v_1) = 1; f(v_2) = 3; f(v_3) = 5; f(v_4) = 7; f(v_5) = 9; f(v_6) = 11$ and

$f(u_1) = 2; f(u_2) = 4; f(u_3) = 6; f(u_4) = 8; f(u_5) = 10$ *and* $f(w) = 11$

Edge labels are,

$f^*(v_2v_1) = 13; f^*(v_3v_2) = 49; f^*(v_4v_3) = 109; f^*(v_5v_4) = 193; f^*(v_6v_5) = 301$ and

$f^*(u_1v_1) = 4; f^*(u_2v_2) = 19; f^*(u_3v_3) = 46; f^*(u_4v_4) = 85; f^*(u_5v_5) = 134$

Hence the edges are distinct.

Remark:

In the similar manner, we can see the cube difference mean labeling of G obtained by joining a pendant vertex with a vertex of degree two on both ends of a comb graph.

We have to prove the following Theorem.

Theorem 4

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both ends of a comb graph. Then G is Cube Difference Mean Labeling Graph.

Proof:

Let the vertex set of a comb be

$V(G) = \{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_n\}$

Let us take P_n as$\{v_1, v_2, \dots, v_n\}$ and join a vertex u_i to v_i for $1 \le i \le n$

Let G be graph obtained by joining a pendant vertex v to v_n and v_1 to u

And the edges are $E(G) = \{a, b, e_i, 1 \le i \le n-1; e_i', 1 \le i \le n$

Which are denoted as in Fig(1.7)

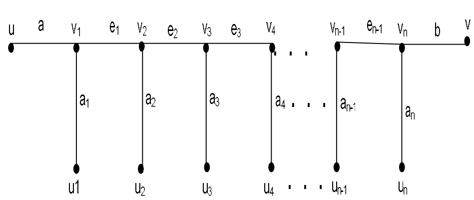

Fig.1.7

Define the function $f: V(G) \to \{1,2,3, \dots, q+1\}$ as follows

$f(u_i) = 2i + 1$ for $1 \le i \le n$

$f(v_i) = 2i$ for $1 \le i \le n$

$f(u) = 1$

$f(v) = 2(n + 1)$

Induced edge labeling function $f^*(uv) = E(G) \to \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$

is defined by

$f^*(v_{i+1}v_i) = \lceil 12i^2 + 12i + 4 \rceil$ for $1 \le i \le n$-1

$f^*(u_iv_i) = \left\lceil \frac{12i^2+6i+1}{2} \right\rceil$ for $1 \le i \le n$

$f^*(v_iu) = \left\lceil \frac{8i^3-1}{2} \right\rceil$ for i=1

$f^*(vv_n) = \lceil 12n^2 + 12n + 4 \rceil$

Clearly

$f^*(e_i) = \lceil 12i^2 + 12i + 4 \rceil$ for $1 \le i \le n$-1

$f^*(e_i') = \left\lceil \frac{12i^2+6i+1}{2} \right\rceil$ for $1 \le i \le n$

$f^*(a) = \left\lceil \frac{8i^3-1}{2} \right\rceil$ for i=1

$f^*(b) = \lceil 12n^2 + 12n + 4 \rceil$

Hence the edge labels are distinct.

This gives Cube Difference Mean Labeling Graph.

Example 4.1

Cube difference mean labeling G with 10 vertices 9 edges is shown below:

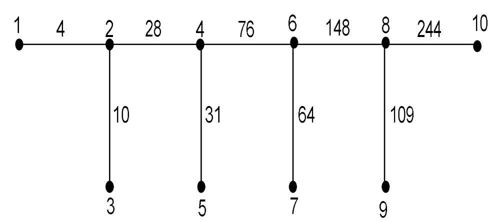

Fig.1.8

Here the vertex labels are,

$f(v_1) = 2; f(v_2) = 4; f(v_3) = 6; f(v_4) = 8,$

$f(u) = 9; f(v) = 10$,

$f(u_1) = 3; f(u_2) = 5; f(u_3) = 7; f(u_4) = 9.$

Edge labels are,

$f^*(a) = 4; f^*(b) = 244; f^*(e_1) = 28; f^*(e_2) = 76; f^*(e_3) = 148$ and

$f^*(e_1') = 10; f^*(e_2') = 31; f^*(e_3') = 64; f^*(e_4') = 109;$

Hence the edges are distinct.

Thus the above graph is Cube Difference Mean Labeling Graph.

Theorem 5:

The Y-Tree Y_n is a Cube Difference Mean Labeling Graph

Proof:

Let $\{v_1, v_2, \ldots, v_n\}$ vertices of Y_n and $\{e_2, \ldots, e_{n-1}\}$ be the edges of Y_n.

Which are denoted as in Fig(1.9)

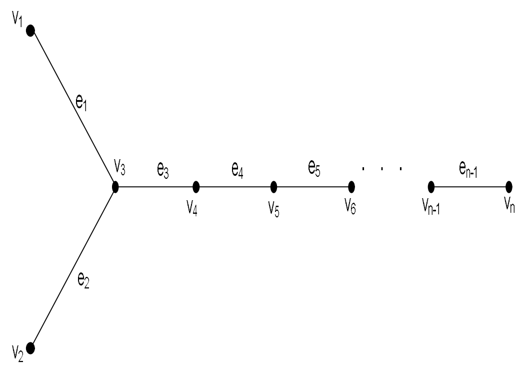

Fig.1.9

Define the function, $f: V(G) \to \{1,2,3,\ldots,q+1\}$ as follows

$f(v_i) = i$ for $1 \le i \le n$

and the induced edge labeling function $f^*(uv) = E(G) \to \{a/1 \le a \le N \text{ and } a \equiv 1(mod3)\}$ is defined by $f^*(uv) = \left\lceil \frac{[f(u)]^3 - [f(v)]^3}{2} \right\rceil$

Now,

$f^*(v_{i+2}v_i) = \lceil 3i^2 + 6i + 4 \rceil, i = 1$

$f^*(v_{i+1}v_i) = \left\lceil \frac{3i^2+3i+1}{2} \right\rceil$ for $2 \le i \le n-1$

Clearly

$f^*(v_{n+2}v_n) = \lceil 3n^2 + 6n + 4 \rceil$

$f^*(v_n v_{n-1}) = \left\lceil \frac{3n^2-3n+1}{2} \right\rceil$

Hence the edge labels are distinct.

Thus Y-Tree Y_n is a Cube Difference Mean Labeling Graph.

Example 5.1

The following example for Y-Tree Y_7 is cube difference mean labeling graph.

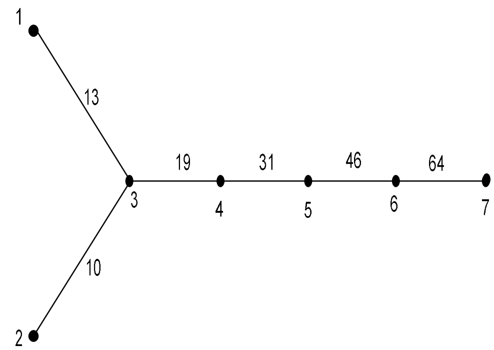

Fig.1.10

Here the vertex labels are,

$f(v_1) = 1; f(v_2) = 2; f(v_3) = 3; f(v_4) = 4; f(v_5) = 5; f(v_6) = 6;\ f(v_7) = 7$ and

The edge labels are,

$f^*(v_3v_1) = 13; f^*(v_3v_2) = 10; f^*(v_4v_3) = 19;$

$f^*(v_5v_4) = 31; f^*(v_6v_5) = 46; f^*(v_7v_6) = 64$

Hence the edge labeling of Y_7 are distinct.

2.REFERENCES

Ajitha, V. ,Arumugam, S. and Germina, K.A. 2006On square sum graphs AKCE J.Graphs, Combin, 6: 1-10.

Amuda, . G. **and** Meena S.**2015.**Cube Difference Labeling of some cycle related graphs, International journal of Innovative science, engineering & Technology, Vol.2 Issue 1

Gallian, . J.A. 2003.A dynamic survey of graph labeling. The Electronic Journal of Combinatorics

Gayathri ,B. and Prakash ,V. 2015.One Modulo Three Mean Labeling of some family of trees, International Journal of Engineering Science, Advanced computing and Bio-Technology Vol.6. No.4. pp 141-161

Gowri**,** R. and Vembarasi, G. **2017.**Root cube mean labeling of graphs, International Journal of Engineering Science, Advanced computing and Bio-Technology Vol.8. No.4., pp 248-255**.**

Harary, F. 1972.Graph theory,Addison Wesley, Massachusetts, Sandhya, S.S. and Somasundaram, S. 2013."Harmonic Mean Labeling or some special graphs" Graph", International Journal of Mathematical Research. ISSN 0976-5840 Volume 7 pp 125-134.pp, 55-64.

Sandhya, S.S. Somasundaram, S. and Anusa, S. 2015." Some Results On Root Square Mean Graph", International Journal of Mathematical Research. ISSN 0976-5840 Volume 7, Number 2 pp 125-134.

Shiama, J.2013 Cube difference labeling of Some graphs International journal of Engineering Science and innovative Technology, Vol.2(6), November 2013.

Somasundaram, S. and R. Ponraj.2003. Mean Labeling of graphs, National Academy Science Letters Vol:26, 210-213.

Somasundaram,. S. and Ponraj, R.2003.Some results on Mean graphs, Pure and Applied. Mathematic Sciences, Vol: 58, 29-35. 1-10.

*****